The calculations performed show that statistical methods are a reliable means for determining the wave parameters of two-phase, ascending, film flow.

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LONGITUDINAL DIFFUSION OF AN IMPURITY IN A PIPE WITH PERMEABLE WALLS

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The problem of the spread of an impurity in a stream with fluid injection through the pipe walls is analyzed.

The theory of convective longitudinal diffusion of a passive impurity in laminar and turbulent streams during fluid flow in pipes with impermeable walls was developed by Taylor [1, 2]. In [3] the solution of this problem was constructed by the method of successive approximations; in this way it was shown that Taylor's model is asymptotically valid as $t \to \infty$. In the present report the solution of the diffusion equation is obtained for long times in the presence of fluid injection through permeable pipe walls.

The distribution of a passive impurity in laminar and turbulent modes of flow in flat and round pipes without allowance for diffusion in the longitudinal direction is described by the equation

$$\frac{\partial c}{\partial t}+u_x\frac{\partial c}{\partial x}+u_r\frac{\partial c}{\partial r}=\frac{D}{r^\alpha}\frac{\partial}{\partial r}\left(r^\alpha\gamma(r)\frac{\partial c}{\partial r}\right). \tag{1}$$

For hydrodynamically stabilized flow in the presence of injection the longitudinal and radial velocity components can be expressed through one function $F(\eta;\ \mathrm{Re}_V)$ [4, 5]:

$$u_x=\left(U_0+\frac{2^\alpha V x}{r_0}\right)\frac{F'(\eta)}{(2\eta)^\alpha},\quad u_r=-V\frac{F(\eta)}{\eta^\alpha}. \tag{2}$$

With allowance for (2), Eq. (1) takes the dimensionless form

$$\frac{\partial c}{\partial\tau}+\frac{\mathrm{Pe}}{2}\frac{\partial c}{\partial X}+\frac{\mathrm{Pe}}{2}\left(\frac{F'(\eta)}{(2\eta)^\alpha}-1\right)\frac{\partial c}{\partial X}-\frac{P}{2}\frac{F(\eta)}{\eta^\alpha}\frac{\partial c}{\partial\eta}=\frac{1}{\eta^\alpha}\frac{\partial}{\partial\eta}\left(\eta^\alpha\gamma(\eta)\frac{\partial c}{\partial\eta}\right). \tag{3}$$

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Here

$$\tau = \frac{tD}{r_0^2}; \quad \eta = \frac{r}{r_0}; \quad X = \frac{\mathrm{Pe}}{2^\alpha P} \ln\left(1 + \frac{2^\alpha V x}{r_0 U_0}\right); \quad \mathrm{Pe} = \frac{2 r_0 U_0}{D}; \quad P = \frac{2 r_0 V}{D}.$$

By analogy with the problem of the diffusion of an impurity in a pipe with impermeable walls, we construct the approximate solution of Eq. (3) by setting

$$\frac{\partial c}{\partial \tau} + \frac{\mathrm{Pe}}{2} \frac{\partial c}{\partial X} = 0, \quad \frac{\partial c}{\partial X} = \frac{\partial \bar{c}}{\partial X} \tag{4}$$

in it, where $\bar{c} = \int_0^1 (2\eta)^\alpha c d\eta$ is the average concentration of the impurity over a pipe cross section.

With allowance for (4), we obtain

$$\frac{\partial}{\partial \eta}\left(\eta^\alpha \gamma(\eta) \frac{\partial c}{\partial \eta}\right) + \frac{P}{2} \frac{F(\eta)}{\eta^\alpha} \frac{\partial c}{\partial \eta} = \frac{\mathrm{Pe}}{2}\left(\frac{F'(\eta)}{2^\alpha} - \eta^\alpha\right) \frac{\partial \bar{c}}{\partial X}. \tag{5}$$

Equation (5) reflects the balance of convective transfer in the longitudinal and transverse directions and of transverse diffusion of the impurity. The boundary conditions for (5) will be

$$\left(\frac{\partial c}{\partial \eta}\right)_{\eta=0} = 0, \quad \left(\frac{\partial c}{\partial \eta}\right)_{\eta=1} - \frac{P}{2}(c_m - c_w) = 0, \tag{6}$$

where $c_m = \int_0^1 F' c d\eta$ is the average-mass concentration; c_w is the concentration of the impurity at the wall.

The second condition of (6) expresses the absence of transfer of the impurity through the surface of the pipe, since in the case of injection the total flux of the transferred substance is equal to the sum of the diffusional and convective fluxes.

The solution of Eq. (5) with the boundary conditions (6) has the form

$$c = c(\eta = 0) + \frac{\mathrm{Pe}}{2} \frac{\partial \bar{c}}{\partial X} \int_0^\eta \frac{d\xi}{\xi^\alpha \gamma} \exp\left(-\frac{P}{2} \int_0^\xi \frac{F d\xi}{\xi^\alpha \gamma}\right) \int_0^\xi \left(\frac{F'}{2^\alpha} - \xi^\alpha\right) \exp\left(\frac{P}{2} \int_0^\xi \frac{F d\lambda}{\lambda^\alpha \gamma}\right) d\xi. \tag{7}$$

We multiply (7) by $(2\eta)^\alpha$ and integrate in the limits from zero to one with allowance for (6):

$$\frac{\partial \bar{c}}{\partial \tau} + \frac{\mathrm{Pe}}{2} \frac{\partial \bar{c}}{\partial X} + \frac{\mathrm{Pe}}{2} \frac{\partial (c_m - \bar{c})}{\partial X} = 0. \tag{8}$$

Equation (8) describes the scattering of the impurity relative to a cross section moving with the average stream velocity $U = U_0 + 2^\alpha V x / r_0$. Determining c_m in (8) from the solution (7), we obtain

$$\frac{\partial \bar{c}}{\partial \tau} + \frac{\mathrm{Pe}}{2} \frac{\partial \bar{c}}{\partial X} = \frac{D_c}{D} \frac{\partial^2 \bar{c}}{\partial X^2}, \tag{9}$$

where D_c is the coefficient of convective longitudinal diffusion,

$$\frac{D_c}{D} = \frac{\mathrm{Pe}^2}{4} \int_0^1 \frac{(F - \eta^{1+\alpha})}{\eta^\alpha \gamma} \exp\left(-\frac{P}{2} \int_0^\eta \frac{F d\xi}{\xi^\alpha \gamma}\right) \int_0^\eta \left(\frac{F'}{2^\alpha} - \xi^\alpha\right) \exp\left(\frac{P}{2} \int_0^\xi \frac{F d\xi}{\xi^\alpha \gamma}\right) d\xi d\eta. \tag{10}$$

Thus, the longitudinal diffusion of an impurity in a pipe with injection is described by the diffusion equation (9), which has exactly the same form as for a pipe with impermeable walls, while the coefficient of convective diffusion is determined by Eq. (10).

At small values of the injection parameter P we get from (10)

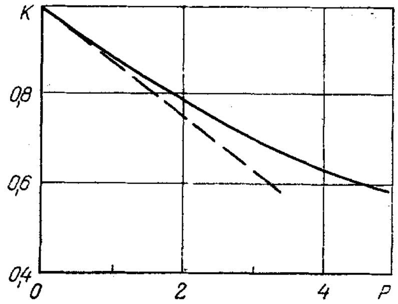

Fig. 1. Dependence of coefficient of convective diffusion on injection parameter.

$$\frac{D_c}{D} = \frac{\mathrm{Pe}^2}{4}\left[\int_0^1 \frac{(F-\eta^{1+\alpha})^2 d\eta}{(2\eta)^\alpha \gamma} - \frac{P}{2^{1+\alpha}}\int_0^1 \frac{(F-\eta^{1+\alpha})}{\eta^\alpha \gamma} \int_0^\eta \frac{F}{\xi^\alpha \gamma}(F-\xi^{1+\alpha})\, d\xi d\eta + O(P^2)\right]. \tag{11}$$

The first term in (11) corresponds to Taylor's solution, while the second determines the correction due to injection.

As an example, let us calculate D_c for laminar flow ($\gamma = 1$) in a round pipe ($\alpha = 1$) at large values of the diffusional Prandtl number $\mathrm{Pr}_D \gg 1$, when $\mathrm{Re}_V \ll 1$ and the hydrodynamics of the flow is described by the Poiseuille function [5]

$$F = 2\eta^2 - \eta^4. \tag{12}$$

In this case from (11) we have

$$\frac{D_c}{D} = \frac{\mathrm{Pe}^2}{192}\left(1 - \frac{P}{8}\right). \tag{13}$$

In Fig. 1, the solid line shows the dependence of $K = D_c/D_{c0}$ ($D_{c0}/D = \mathrm{Pe}/192$) on the parameter P, calculated from Eq. (10) with allowance for (12); the dashed straight line was constructed in accordance with (13). As seen from the figure, the coefficient of longitudinal diffusion decreases with an increase in the injection intensity.

A decrease in the coefficient of convective diffusion is a favorable circumstance for the application of the method of a thermal marker (an optical flag) in gasdynamic visual investigations of flows with injection. In addition, the result obtained confirms the efficiency of the use of injection for purposes of protection of the channel walls from corrosive impurities.

NOTATION

t, time; X, r, longitudinal and radial coordinates; c, concentration of impurity; r_0, pipe radius; D, coefficient of molecular diffusion; D_t, coefficient of turbulent diffusion; ν, coefficient of kinematic viscosity; U_0, velocity in entrance cross section; V, injection velocity; $\mathrm{Pr}_D = \nu/D$; $\gamma = 1 + D_t/D$; $\mathrm{Re}_V = 2r_0V/\nu$; $\alpha = 0$ corresponds to a flat pipe and $\alpha = 1$ to a round pipe.

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